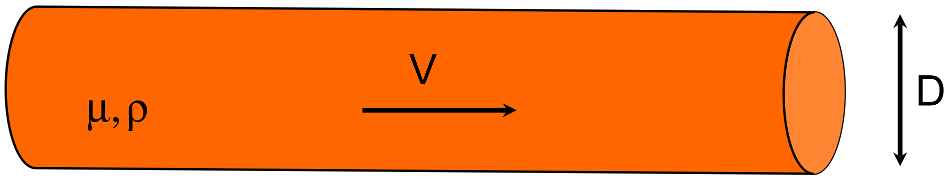
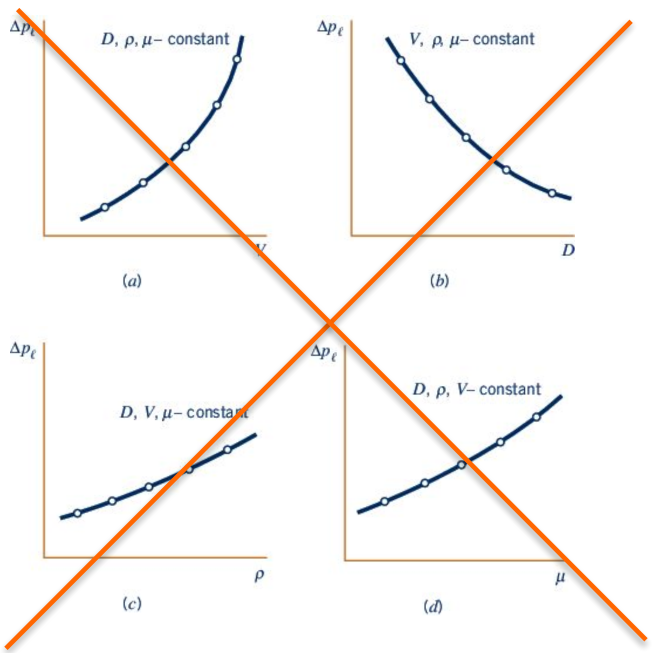
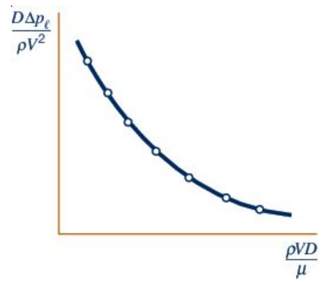
Dimensional analysis



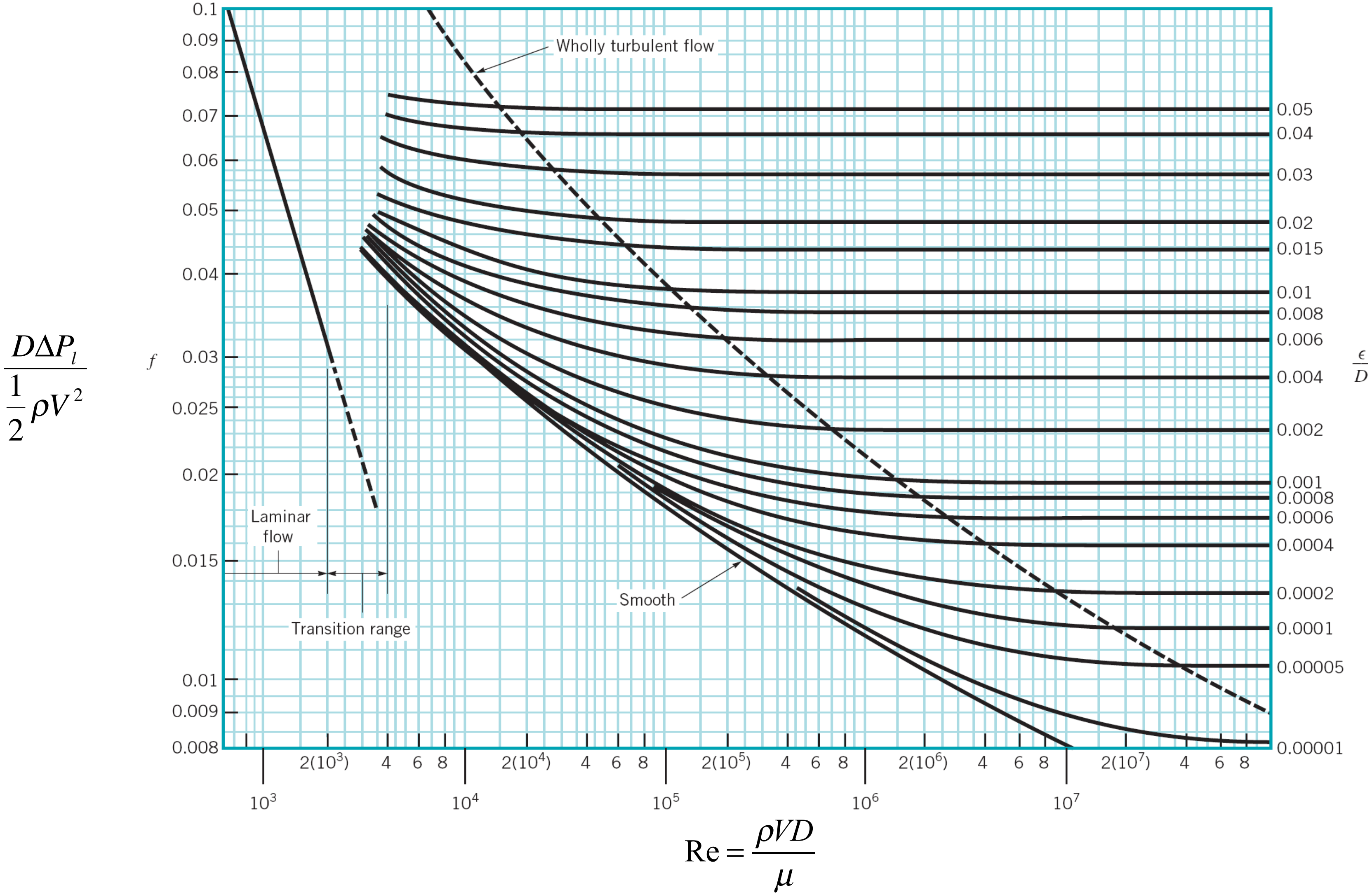
$$\Delta P_l = f(D, \rho, \mu, V)$$



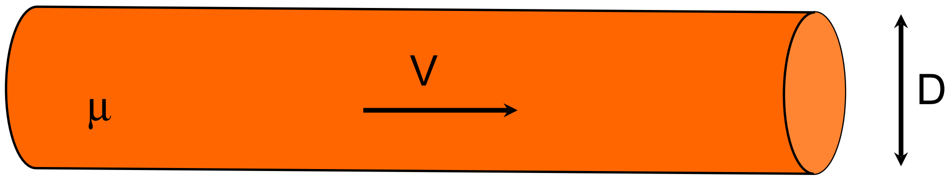


$$\frac{D\Delta P_l}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}\right)$$

Moody chart (flow in pipes)



Example: viscous flow in a pipe



$$\Delta p_l = f(D, \mu, V)$$

$$\Delta p_l \doteq F L^{-2} L^{-1} \doteq F L^{-3}$$

$$D \doteq L$$

$$\mu \doteq F T L^{-2}$$

$$V \doteq L T^{-1}$$

4 variables − 3 reference dimensions = 1 π-term

$\Delta p_l, D, \mu, V$ $\qquad$ $F\ L\ T$

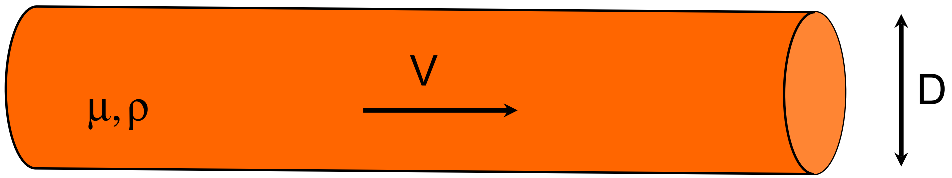
$$\frac{\Delta p_l D^2}{\mu V} \qquad \frac{\cancel{F}\ \cancel{L^{-3}}\ \cancel{L^{2}}}{\cancel{F}\ \cancel{T}\ \cancel{L^{-2}}\ \ L\ \cancel{T^{-1}}}$$

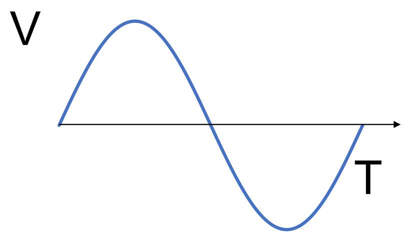
Hence, $\dfrac{\Delta p_l \cdot D^2}{\mu V} = K$ (constant) $\Rightarrow \dfrac{\Delta p}{l} = K \cdot \dfrac{\mu}{D^2} V = K \dfrac{\mu}{4R^2} \cdot \dfrac{Q}{\pi R^2}$

$$\Rightarrow \underline{\underline{\Delta p = \frac{K}{4} \cdot \frac{\mu l}{\pi R^4} Q = K' \frac{\mu l}{\pi R^4} Q}}$$

Poiseuille's law: $\Delta p = \dfrac{8 \mu l}{\pi R^4} Q$

Dimensional analysis for pulsatile flow in a straight rigid tube





Periodic (harmonic) flow with $\omega = \dfrac{2\pi}{T}$

$$\Delta p_l = f(D, \rho, \mu, V, \omega)$$

$$\Delta p_l \doteq F L^{-3}$$
$$D \doteq L$$
$$\rho \doteq M L^{-3} \doteq F T^{2} L^{-4}$$
$$\mu \doteq F T L^{-2}$$
$$V \doteq L T^{-1}$$
$$\omega \doteq T^{-1}$$

6 variables
−
3 reference dimensions
=
3 π-terms

By simple inspection:

$$\Pi_1 = \frac{\Delta p_l \cdot D}{\rho V^2} \qquad \Pi_2 = \frac{\rho V D}{\mu} \qquad \Pi_3 = \frac{\omega D}{V}$$

$$\therefore \quad \frac{\Delta p_l \cdot D}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}, \frac{\omega D}{V}\right)$$

or

$$\frac{\Delta p_l \cdot D}{\rho V^2} = \phi'\left(\frac{\rho V D}{\mu}, \frac{\omega \rho D^2}{\mu}\right)$$

Hence,

$$\frac{\Delta p_l \cdot D}{\rho V^2} = \phi''(Re, \alpha)$$

Define the Womersley parameter, α, as

$$\alpha^2 = \frac{\omega \rho R^2}{\mu} \Rightarrow \alpha = R\sqrt{\frac{\omega \rho}{\mu}}$$